WHAT MIGHT GRACE CONTRIBUTE TO STUDIES OF POST GLACIAL REBOUND?

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Abstract. The NASA/DLR satellite gravity mission GRACE, launched in March, 2002, will map the Earth's gravity field at scales of a few hundred km and greater, every 30 days for five years. These data can be used to solve for time-variations in the gravity field with unprecedented accuracy and resolution. One of the many scientific problems that can be addressed with these time-variable gravity estimates, is post glacial rebound (PGR): the viscous adjustment of the solid Earth in response to the deglaciation of the Earth's surface following the last ice age.

In this paper we examine the expected sensitivity of the GRACE measurements to the PGR signal, and explore the accuracy with which the PGR signal can be separated from other secular gravity signals. We do this by constructing synthetic GRACE data that include contributions from a PGR model as well as from a number of other geophysical processes, and then looking to see how well the PGR model can be recovered from those synthetic data. We conclude that the availability of GRACE data should result in improved estimates of the Earth's viscosity profile.

1. Introduction

GRACE, jointly sponsored by NASA and the Deutsches Zentrum für Luft- und Raumfahrt (DLR), was launched in March, 2002. It will map the Earth's gravity field with unprecedented accuracy and resolution every 30 days during its 5-year lifetime. This will permit monthly variations in gravity to be determined down to scales of a few hundred kilometers and larger. These gravity variations can be used to study a variety of processes involving redistribution of mass within the Earth or on its surface. The expected performance of GRACE and various possible applications are described by Dickey et al. (1997) and Wahr et al. (1998).

Among these applications is the use of the GRACE secular gravity signal to constrain models of post glacial rebound (PGR): the viscous adjustment of the solid Earth in response to the removal of the ice loads following the last ice age. PGR studies are useful from a solid Earth perspective, because they provide information about the Earth's viscosity profile. The PGR process is also an error source when interpreting various types of observations relevant to global sea level change, including altimeter estimates of ice sheet thickness variability and tide gauge estimates of the sea level change itself.

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This paper describes a preliminary look at GRACE's ability to recover the PGR signal. A more detailed description can be found in Velicogna and Wahr (2002a). We will focus primarily on the time-variable gravity signal over northern Canada. It will probably be harder for GRACE to constrain the PGR signal over Scandinavia, due to the smaller amplitudes and shorter spatial scales that characterize the signal there. The PGR gravity signals over Greenland and Antarctica are likely to be severely contaminated by the gravity signals from the present-day mass imbalance of the Greenland and Antarctic ice sheets. Methods of combining GRACE and ice sheet altimeter data to separate the PGR and ice sheet signals in those regions are described by Wahr et al. (2000), Velicogna and Wahr (2002b), and Wu et al. (2002).

2. The PGR Signal

A PGR model requires knowledge of the Earth's viscosity profile and of the ice deglaciation history. These are determined by comparing PGR model output and observations. Specific types of observations tend to be more sensitive to particular parameters. For example, the viscosity of the Earth's upper mantle can be especially well constrained using geological observations of past changes in relative sea level, and geodetic observations of present-day crustal uplift. Observations more sensitive to lower mantle viscosity include the present-day free-air gravity and geoid anomalies over Canada and Scandinavia, and secular changes in the Earth's rotation and in the earth's gravity field as determined from satellites.

Past interpretations of the observations most sensitive to lower mantle viscosity have been somewhat ambiguous. The present-day gravity and geoid anomalies may have sizable tectonic contributions, unrelated to PGR. The Earth rotation and time-variable gravity observations could well have significant contributions from other quasi-secular processes, including present-day changes in polar ice.

In this paper, we investigate whether time-variable gravity from GRACE will have the resolution and accuracy needed to separate the PGR effects from the contaminating effects of other secular processes. We do this by constructing simulated GRACE data using output from a PGR model, and then trying able to use those GRACE data to recover the input model used in the simulation.

For our estimate of the deglaciation history we use the global ICE-3G Pleistocene ice model of Tushingham and Peltier (1991), with an additional 90 kyr linear glaciation phase added at the beginning. Our viscosity profile consists of three uniform viscosity layers: the uppermost mantle (between the base of the lithosphere and 400 km depth), the transition zone (between 400 km and 670 km depth), and the lower mantle (between the core-mantle boundary and 670 km depth). The overlying lithosphere and underlying fluid core are assumed to be inviscid. We will simulate results for many viscosity values and lithospheric thicknesses. For our "default" model, we assume a lithospheric thickness of 100 km, uppermost mantle and transition zone viscosities of 1.0×10^{21} Pa s, and a lower mantle viscosity of

TABLE I
Elastic Parameters Used to Compute All PGR Contributions

Layer	Outer radius (R, km)	Density (ρ, kg/m^3)	Shear wave speed (v_s, km/s)
Core	3480	10,925	0.0
Lower Mantle	5701	4,970	6.6
Transition Zone	5971	3,850	5.25
Uppermost Mantle	6271	3,070	4.33
Lithosphere	6371	3,070	4.33

1.0×10^{22} Pa s. For the elastic structure, we assume the five spherical layers are incompressible and homogeneous. Their densities and shear wave velocities are chosen to be reasonably consistent with the structural model PREM (Preliminary Reference Earth Model) of Dziewonski and Anderson (1981), and are given in Table I.

3. Time Variable Gravity from GRACE

It is usual to expand the geoid height, N, as a sum of associated normalized Legendre functions, $\tilde{P}_{lm}$, in the form (see, *e.g.*, Chao and Gross, 1987):

$$N(\theta, \phi) = a \sum_{l=0}^{\infty} \sum_{m=0}^{l} \tilde{P}_{lm}(\cos\theta) \, [C_{lm} \cos(m\phi) + S_{lm} \sin(m\phi)] \,, \tag{1}$$

where θ and ϕ are co-latitude and eastward longitude, and the C_{lm}'s and S_{lm}'s are dimensionless Stokes' coefficients. GRACE measurements will be used to determine the C_{lm}'s and S_{lm}'s up to degree and order (*i.e.* l and m) = 100 every 30 days. For each $\tilde{P}_{lm}$ term in this expansion, the horizontal scale (half-wavelength) is approximately 20,000/l km.

PGR causes secular variations in the C_{lm}'s and S_{lm}'s. But so do other processes. Present-day changes in the Greenland and Antarctic ice sheets pose the most serious problems for using GRACE to learn about PGR. They are likely to be the largest non-PGR sources of secular gravity, and they are located at latitudes similar to PGR latitudes. This has, historically, made it difficult to separate these signals in the time-variable satellite gravity field.

Prior to the launch of GRACE, all satellite estimates of time-variable gravity came from satellite laser ranging (SLR). The SLR secular terms consist of a few zonal coefficients: i.e. C_{l0}'s for $l = 2$ up to maybe $l = 6$ (see, for example, Cheng et al., 1997). This is not enough coefficients to allow for easy separation of

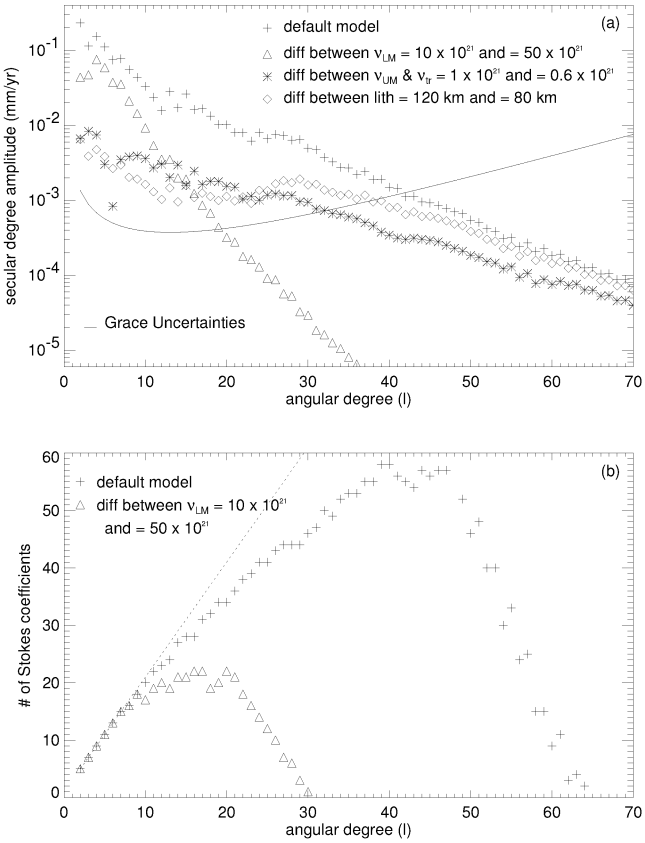

Figure 1. (a) Predictions of degree amplitudes for the secular change in the geoid, as a function of angular degree l. Shown are predictions for the default PGR model (plus signs), and for the differences between the default PGR model and three other plausible PGR models (ν_{UM}, ν_{LM}, and ν_{tr} are the viscosities, in Pa s, of the uppermost mantle, lower mantle, and transition zone; and lith is the lithospheric thickness). Also shown (solid line) are the degree amplitudes of the expected secular Grace measurement errors. (b) The number of Stokes' coefficients at a given value of l where the amplitude of the predicted PGR signal is larger than the expected secular GRACE measurement error. Shown are the results for the default PGR model, and for the difference between the PGR signals for two plausible lower mantle viscosity values. The dotted line shows the total number of Stoke's coefficients as a function of l. All together, there are a total of 2193 Stokes' coefficients for the default PGR signal, and 424 coefficients for the difference between the two PGR signals, that rise above the expected GRACE errors.

PGR from the competing secular signals. For example, the difference in longitude between Hudson Bay (the center of the largest Pleistocene ice sheet) and Greenland is about 40°. Separation of these two signals would thus require angular orders up to about $m = 360°/40° = 9$. Zonal coefficients ($m = 0$) alone, do not permit this separation.

GRACE, though, will have considerably higher spatial resolution and accuracy. Figure 3a compares the secular degree amplitudes, defined as

$$N_l = a \sqrt{\sum_{m=0}^{l} \left(C_{lm}^2 + S_{lm}^2\right)} \quad , \tag{2}$$

of the default PGR geoid signal (the plus signs), with the expected degree amplitudes of the secular GRACE measurement errors as estimated by B. Thomas and M. Watkins [personal communication]. The results in Figure 3a show the PGR signal should be larger than the GRACE measurement errors for degrees of about 40 and smaller. Since there are $2l + 1$ values of m for every l, this suggests there should be about 40^2 Stokes' coefficients where the PGR signal is larger than the secular GRACE errors. In fact, there are 2193 such coefficients, shown in Figure 3b (the plus signs) as a function of l.

Given this high sensitivity of GRACE, it is natural to also consider GRACE's sensitivity to the effects of different viscosity profiles. The other symbols in Figure 3a show the degree amplitudes of the differences between pairs of PGR models. For each of the three PGR pairs considered in this figure, one model is the default model, and the other is obtained from the default model by changing either the lower mantle viscosity, the uppermost mantle and transition zone viscosity, or the lithospheric thickness. Note that for low angular degrees, where GRACE is most accurate, the PGR results are most sensitive to the lower mantle viscosity. The triangles in Figure 3b show there are a large number of Stokes' coefficients (a total of 424) where the difference between PGR models for two different but plausible lower mantle viscosities rise above the expected GRACE errors.

4. GRACE Simulations

The results shown in Figure 3 are encouraging, because they imply that the GRACE measurement accuracy should easily be good enough for GRACE to be sensitive to the PGR signal. But those results say nothing about the problem of separating the PGR signal from other gravity signals that appear to be secular over the five year lifetime of GRACE. To address that issue, we construct a suite of plausible, simulated GRACE data sets that include time-variable gravity signals from a number of sources, including PGR. We look to see how well we can recover the PGR signal from each of those simulated data sets.

Each simulated GRACE data set is in the form of five years of monthly-averaged Stokes' coefficients. In those coefficients we include a realization of the GRACE measurement errors estimated by Thomas and Watkins, as well as the predicted gravity signals from PGR (using our default model); from spatially-uniform, secular ice mass changes over Antarctica and Greenland (estimated assuming sea level rise contributions from Antarctica and Greenland that are within the range of 1 ±

1 mm/yr and 0.1 ± 0.15 mm/yr, respectively – numbers consistent with the mass balance estimates summarized by Church et al. (2001); from a 1.0 mm/yr non-steric global sea level rise; from month-to-month variations in global, continental water storage (provided by C. Milly and K. Dunne, personal communication); from month-to-month variations in sea floor pressure caused by changes in oceanic circulation (estimated by M. Molenaar and F. Bryan, personal communication) from the POP ocean general circulation model (Dukowicz and Smith, 1994); and from errors in the GRACE atmospheric pressure corrections over land, estimated by taking the differences between ECMWF (European Centre for Medium-Range Weather Forcasts, 1995) and NCEP (Kalnay et al., 1996) pressure fields, divided by $\sqrt{2}$.

Once we have constructed the simulated GRACE data, we use two methods to get an idea of how well the default PGR model could be recovered from the data. Both methods fall short of the full inversion that people will undoubtedly use with the real GRACE data. But they establish some initial expectations for how well the PGR signal can be separated from other sources of secular gravity.

4.1. GRACE Recovery of PGR: Method 1

As a first attempt to look at the effects of contamination, we simply least squares fit the default PGR signal to the simulated GRACE data. In the absence of contamination, and if there were no GRACE errors, the fit parameter would equal 1.

Figure 4.1a shows the secular geoid change inferred from one of the simulated GRACE data sets; where the sea level rise contributions from Antarctica and Greenland are equal to 1 mm/yr and 0.1 mm/yr, respectively. Figure 4.1a thus represents a secular geoid signal that might be extracted from the GRACE data. Note that the Canadian PGR signal (Figure 4.1b) is clearly evident in the total signal (Figure 4.1a).

The fact that this signal can be so clearly identified by simply looking at Figure 4.1a, suggests that GRACE will be able to deliver this signal reasonably free of contamination from the other processes. To obtain a rough estimate of the level of contamination, we truncate the total GRACE result and the PGR signal to the region inside the box. We remove the spatial mean over this region from both signals, and least-squares fit the residual PGR signal to the total signal over this region.

For the parameters used to construct the simulation shown in Figure 4.1a, we obtain a fit parameter of 0.96. When we repeat this procedure for other simulated GRACE data sets, obtained by varying the present-day Antarctic and Greenland ice thickness rates, we obtain fit parameters that vary between about 0.94 and 0.96, which we crudely interpret as implying that GRACE would be able to recover the Canadian PGR signal to an accuracy of about 4–6%. We find we can obtain somewhat better results when we simultaneously fit other parameters to the data (e.g. present-day Antarctic and Greenland mass changes, global sea level change).

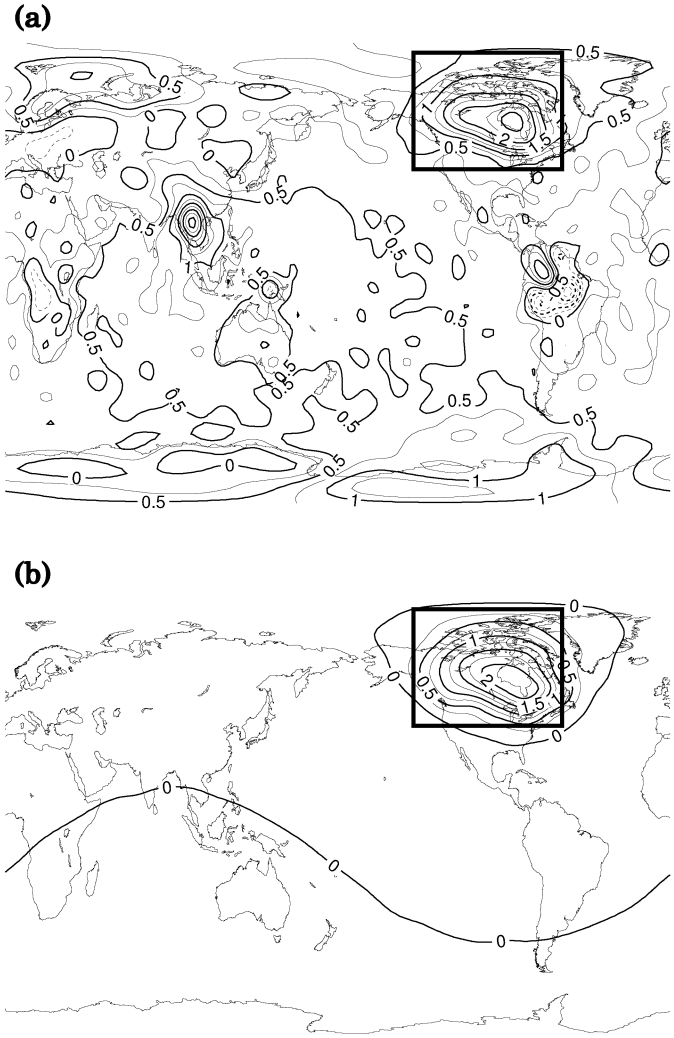

Figure 2. The secular change in the simulated GRACE geoid after five years. Obtained by adding the default PGR signal to predictions of the secular geoid contributions from continental water storage, changes in Greenland and Antarctic ice, the distribution of water in the oceans (including an assumed, non-steric global sea level rise), errors in the GRACE atmospheric pressure corrections, and a realization of the GRACE observational errors. (b) The predicted contribution to the secular geoid change from the default PGR model caused by Canadian ice, alone.

4.2. GRACE Recovery of PGR: Method 2

The simple method described in Section 4.1 does not address the larger issue of estimating the improvement we can expect in our knowledge of the Earth's viscosity profile as a result of GRACE. A full resolution of that issue would require a simultaneous inversion of simulated GRACE data and other PGR-related observations,

and should also take into account the likely uncertainties in the ice deglaciation model. Instead, we here adopt the following, more modest approach.

We generate secular gravity fields for a large number of PGR models. For each model we use ice model ICE-3G and the elastic structure given in Table 1, but we vary the lithospheric thickness and the viscosity values in the uppermost mantle, the transition zone, and the lower mantle. We remove these PGR secular gravity fields, one at a time, from the simulated secular GRACE gravity field shown in Figure 4.1a. That simulated field was constructed using our default PGR model. After removing a PGR model, we compute the generalized prediction error, R^2, defined as:

$$R^2 = \sum_{l=2}^{l_{\max}} \sum_{m=0}^{l} \frac{\left[\dot{C}_{lm}^{\text{sim}} - \dot{C}_{lm}^{\text{mod}}\right]^2 + \left[\dot{S}_{lm}^{\text{sim}} - \dot{S}_{lm}^{\text{mod}}\right]^2}{\sigma^2(l)} \Bigg/ \sum_{l=2}^{l_{\max}} \frac{(2l+1)}{\sigma^2(l)} \tag{3}$$

where the $\sigma(l)$ are the degree amplitudes of the secular GRACE measurement errors from Thomas and Watkins, the $\dot{C}_{lm}^{\text{sim}}$ and $\dot{S}_{lm}^{\text{sim}}$ are the Stokes' coefficient rates for the simulated GRACE data, and the $\dot{C}_{lm}^{\text{mod}}$ and $\dot{S}_{lm}^{\text{mod}}$ are the PGR coefficient rates predicted by the Earth model removed from the simulated data. The PGR model that minimizes the generalized prediction error (3) is interpreted as the preferred model. We compare the viscosity values used to construct that preferred model, with the viscosity values of the default model used to generate the simulated data.

After finding $R^2_{\min}$, i.e., the generalized prediction error for the model that best fits the measurements, we estimate confidence intervals via the likelihood ratio method (Beck and Arnold, 1977). If the errors are jointly normal, zero mean, and uncorrelated, then the confidence region with the probability α of containing the solution, corresponds to the volume of the model parameter space for which

$$R^2 \leq R^2_{\min}\left[1 + \frac{M}{n-M}\mathcal{F}_{\alpha}^{-1}(M, n-M)\right], \tag{4}$$

where M is the number of model parameters, n is the number of measurements, and $\mathcal{F}^{-1}$ is the inverse of the F cumulative distribution function.

A complicating factor is that the PGR models include perturbations to the gravity field over Antarctica and Greenland, since ICE-3G includes Holocene deglaciation of those regions. The effects of present-day changes in Antarctic and Greenland ice included in the simulated GRACE data, tend to look similar in spatial pattern to those PGR contributions. Thus, as in Section 4.1, we here address the more restricted problem of focusing on the PGR signal only over Canada. We fit and remove, from $\dot{C}_{lm}^{\text{sim}} - \dot{C}_{lm}^{\text{mod}}$ and $\dot{S}_{lm}^{\text{sim}} - \dot{S}_{lm}^{\text{mod}}$, the secular gravitational effects of uniform ice changes in Antarctica and Greenland, and a uniform non-steric sea level change. We do this prior to constructing R^2.

The R^2 results are shown in Figure 4.2a-c for PGR models in which we vary pairs of parameters (e.g., uppermost mantle viscosity and transition zone viscosity, etc.) in a grid-search fashion, keeping the remaining parameters fixed to the default

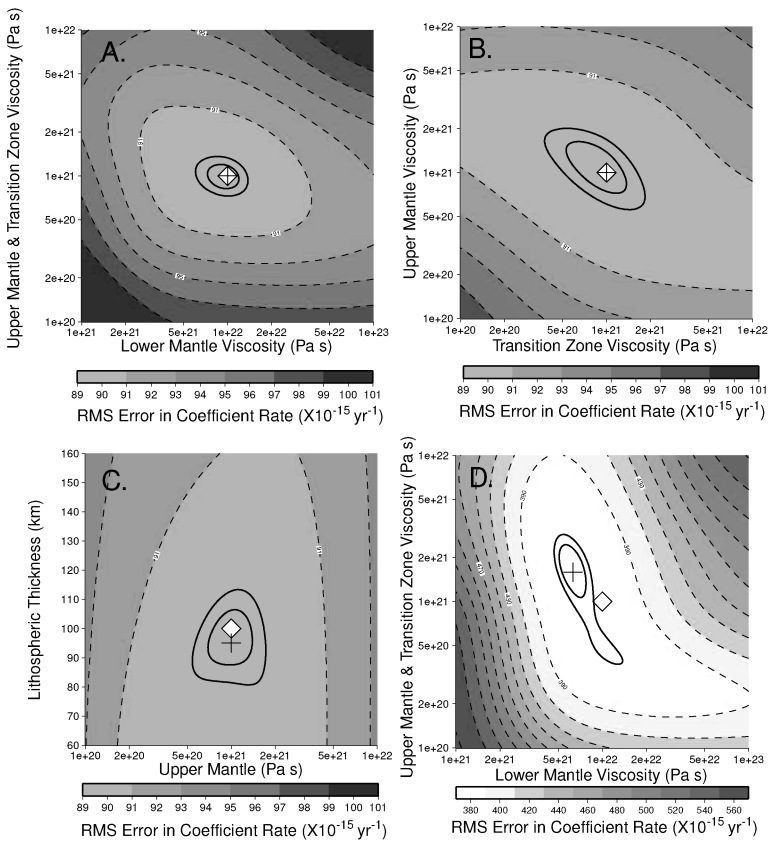

Figure 3. Square root of the generalized prediction error as defined in equation (3), for (a) lower mantle viscosity and uppermost mantle/transition zone viscosity, (b) uppermost mantle and transition zone viscosities, (c) lithospheric thickness and uppermost mantle viscosity. Panel (d) is similar to panel (a), except that a different ice history (ICE-4G) is used to calculate models in the parameter grid search than is used to generate the simulated data (ICE-3G).

values. We include one scenario (Figure 4.2a) in which the uppermost mantle and transition zone viscosities are set to the same value, which is then varied simultaneously with the lower mantle viscosity. Confidence intervals are estimated using $M = 2$ in equation (4). The viscosity values that produce the minimum prediction error are at the centers of the crosses, and the values used for the default model are at the centers of the diamonds. The solid contours represent the 65% and 95% confidence contours.

In general, the values that produce the minimum prediction errors agree well with the default values. The 95% confidence limits on the lower mantle viscosity are between about 6×10^{21} and 14×10^{21} Pa s. Since the default value is 10×10^{21} Pa s, we infer that GRACE is capable of inferring the lower mantle viscosity to about $\pm$ 40%. For the combined uppermost mantle/transition zone viscosity shown

in Figure 4.2a, the 95% confidence interval extends between 0.75×10^{21} and 1.3×10^{21} Pa s, which agrees with the default value (1.0×10^{21} Pa s) to within $\pm$ 30%. The results shown in Figure 4.2c suggest that the lithospheric thickness can be recovered to within $\pm$ 15–20% (the 95% confidence interval extends between 80 and 115 km). Figure 4.2b shows that GRACE would have a harder time discriminating between separate viscosities in the uppermost mantle and transition zone, where for each of those parameters the 95% confidence interval includes numbers that are between one half and twice the correct value. This is presumably because there is a rough proportionality between vertical and horizontal scales in the PGR solution, so that resolving these two thin layers requires information at short scales. GRACE will not do as well at short scales as at longer scaless.

The results shown in Figure 4.2a-c assume there are no errors in the ice model used to calculate the PGR signal. Although this assumption is certainly false, its effects are difficult to determine since there is no obvious way to estimate the ice model errors. To get some idea of how large these effects might be, we consider a second ice history model: ICE-4G from Peltier (1994). ICE-4G is an improved model that uses new constraints which were not available when ICE-3G was developed. We generate the synthetic GRACE data using a PGR signal calculated from ICE-3G, but we use ICE-4G to compute the PGR signals used in the grid search.

The results shown in Figure 4.2d, show that the difference between the ice models can have significant effects on the recovered viscosity. The preferred uppermost mantle/transition zone viscosity is now about 1.6×10^{21} Pa s, or 1.6 times the default value. The preferred lower mantle mantle viscosity is 6×10^{21} Pa s, 40% less than the default value. It is of course possible that this analysis underestimates the effects of true uncertainty in the ice model, since all the data used to constrain ICE-3G were also incorporated into ICE-4G. However, this is perhaps the best estimate we can make for the likely uncertainties.

5. Discussion and Summary

The results presented in Section 3 show that after five years of operation, GRACE should deliver a large number of secular gravity coefficients (in excess of 2000) with errors that are smaller than the PGR signal. In fact, there should be several hundred coefficients with errors smaller than the differences between plausible PGR models. We conclude that the accuracy of the GRACE measurements will be good enough that it is not likely to be a limiting factor for PGR studies.

Instead, the overriding concern is whether the PGR signal can be separated from other sources of secular gravity. This is partly a question of whether GRACE has sufficient resolution to differentiate between the different spatial patterns of the various signals. But even perfect resolution would not allow the separation of every combination of possible signals. For example, separating the PGR signal over Greenland from the combined effects of present-day changes in Greenland

ice mass and secular changes in the surrounding ocean mass distribution, is not possible without invoking additional assumptions about the nature of those signals [see, for example, Wu et al., 2002].

The results presented in Section 4 address the signal separation issue. They suggest that by using GRACE data alone, and if the ice model were perfectly known, it could conceivably be possible to determine the lower mantle viscosity, with 95% confidence, to $\pm$ 40%. Surprisingly, the results also show that GRACE could determine the combined viscosity of the uppermost mantle/transition zone to even better accuracy: $\pm$ 30%. It would be more difficult to use GRACE to solve for separate viscosities in the uppermost mantle and transition zone.

These results do not fully consider the possible effects of unmodeled complexities in the PGR model. When we applied the method described in Section 4.2, we assumed that the set of PGR models removed from the simulated GRACE data included a model with the correct viscosity profile. In reality, the Earth's viscosity profile could well be far more complex, in both its radial and horizontal stratification, than that assumed for any of the forward models constructed for an analysis of this type. The issues then are whether the best-fitting forward model would have a viscosity profile close to the real viscosity profile; and whether it would be possible to converge to the more realistic model by using the sort of minimization method described in Section 4.2.

Errors in the ice model may be even more of a concern. The results shown in Figure 4.2d give some indication of the effects those errors might have on the inferred viscosity. Still, we find it encouraging that the viscosity values we obtain using the wrong ice model, are still within a factor of two of the correct values.

Ultimately, the use of GRACE data to help determine the viscosity and the ice history, would best be done using a formal inversion procedure. The GRACE data would not be used alone, but would be combined with other geological and geodetic measurements of the PGR process. The results of this paper suggest that the GRACE data should add considerable resolving power to any such inversion.

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